



Estimation of Infiltration Coefficients Based on the Average Infiltration Opportunity Time of the Advance Phase

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Article Info	Abstract
<p>Article history:</p> <p>Received: 28 May 2023 Received in revised form: 9 July 2023 Accepted: 28 September 2023 Published online: 29 September 2023</p> <hr/> <p>DOI: 10.22044/JHWE.2023.13185.1022</p> <p>Keywords Surface irrigation Infiltration coefficients Volume balance Two-point method</p>	<p>The two-point method is the most commonly used method of calculating the coefficients of the Kostiakov-Lewis infiltration relationship, which is based on the volume balance equation. Using the two-point method of Elliott and Walker (EW), the coefficient of the power relation of water advance can be determined by initially utilizing data from the midpoint and endpoint of the field. Subsequently, the coefficients of the infiltration relationship can be determined. In this study, all the advance data were used to determine the power relation coefficients of water advance. Also, in this research (TR), the point where its infiltration opportunity is equal to the average infiltration opportunity time of the advance phase was considered as the midpoint in the two-point method for determining the infiltration coefficients. The relative error index and Root Mean Square Deviation (d_{RMS}) index were used to assess the accuracy of the advanced relationships and infiltration equations derived from TR and EW methods. The results showed that the advanced relationships obtained from the TR method have higher accuracy than the EW method. The average relative error index of the infiltration depth indicated that the infiltration relations obtained from the TR method with an average relative error of 2.4% is more accurate than the EW method.</p>

1. Introduction:

Surface irrigation is a widely used form of irrigation in which water is applied to a field or agricultural land in the form of a thin sheet. This method has been around for centuries, and is still widely used today due to its low cost, simplicity, and flexibility. Despite the development of pressurized irrigation

methods, surface irrigation still holds a large share of the global irrigation market (Furman et al., 2006). These systems usually have low efficiency due to improper design and improper management (Merriam, 1977; Seyedzadeh et al., 2022a). The most important parameter affecting the design and management of these types of systems is

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determining infiltration. Various researchers, including Green and Ampt (1911), Kostiakov (1932), Lewis (1937), Horton (1939), Philip (1957), and Smith (1972) have proposed different equations to mathematically represent soil infiltration. One of the most widely used models is the Kostiakov-Lewis infiltration model, which has the following equation:

$$i = kt^a + f_o t \quad (1)$$

where i is infiltration rate (m); t is intake opportunity time (min); a (dimensionless) and k (m/min ^{a}) are the empirical constant coefficients, and f_o is the basic intake rate of the soil (m/min).

To properly design a surface irrigation system and increase its efficiency, it is essential to determine the infiltration coefficients of the Kostiakov-Lewis relationship. This can be done through field measurements or a combination of field measurements and theoretical calculations. By properly determining the infiltration coefficients, the design of a surface irrigation system can be optimized to increase its efficiency and reduce water losses.

Multiple approaches have been employed to identify the three empirical parameters of the Kostiakov-Lewis equation. Christiansen et al. (1966) used volume balance equation and advance data, Elliott et al. (1983) employed the method of matching dimensionless advance curves, while Sirjani and Wallender (1989) determined the k and a parameters through advance data. Elliott and Walker (1982), Shepard et al. (1993), Walker (2005), Seyedzadeh et al. (2020a), Seyedzadeh et al. (2020b), and Panahi et al. (2022) have all attempted to develop simpler methods for calculating the coefficients. From among these, the Elliott and Walker (1982) two-point method has become widely used due to its simplicity and satisfactory accuracy in estimating infiltration depth. Elliott and Walker (1982) utilized the volume balance approach to determine the coefficients of the Kostiakov-Lewis infiltration equation and

took into consideration the water advance relationship as a power equation.

When infiltration opportunity time is equal in the field, the infiltration coefficients are estimated using curve fitting. However, in surface irrigation, infiltration opportunity time is variable based on the location in the field, so alternative methods have to be employed (Panahi et al., 2021). Elliott and Walker (1982) employed the volume balance method to calculate the coefficients. This method involves calculating the total volume of water that enters the field, which is equal to the sum of the water surface storage and subsurface infiltrated water.

Elliott and Walker (1982) used the two-point method to calculate the value of the constant coefficients of the advance relationship from the water advance information at the mid-point and end-point of the field. Then, they determined the volume of water stored in two stages, from the beginning to the mid-point and to the end-point of the field, by utilizing the calculated advance relationship. Subsequently, they determined the volume of infiltrated water by comparing the total volume of water entering the field and the volume of surface stored water and calculated the coefficients of the Kostiakov-Lewis infiltration relationship through the volume balance equation.

The power coefficient of the advance relation (r) is an essential element in calculating the surface water volume stored and the coefficients of the infiltration relation (Emamgholizadeh et al., 2022; Seyedzadeh et al., 2022b). The two-point method determines this coefficient using two static points along the field. However, the soil texture variation along the field may cause the value of r to turn negative (resulting in an unphysical water advance relation), and thus, the coefficients of the infiltration relation will become negative and unphysical (Elliott and Walker, 1982; Seyedzadeh et al., 2019). Studies have shown that since the mid-point is continuously taken into account while defining these coefficients, the information

of this point will be arbitrary and potentially lead to inaccurate outcomes (Seyedzadeh et al., 2020b). Thus, to determine a suitable location for the midpoint in the two-point method, this research will introduce a location for the midpoint based on the average infiltration opportunity time in the advance phase. The values of the coefficient r and the coefficients of the Kostiakov-Lewis infiltration relationship will then be determined and evaluated using the data from this new point and the endpoint.

2. Materials and Methods

2.1. Theory Background

In the two-point method, Elliott and Walker (1982) considered the relationship of water advance along the field as the following exponential relationship:

$$x = pt_x^r \quad (2)$$

where t_x is the time for water advance to x (min); x is the distance of the water front from the upstream end of the field (m); and, r and p are the constant coefficients that are determined empirically.

Using the volume balance method, the volume of infiltrated water (V_x) up to any distance from the beginning of the field (x) can be determined using the following equation:

$$V_x = \sigma_z kt_x^a = \frac{Qt_x}{x} - \sigma_y A_o - \frac{f_o t_x}{r+1} \quad (3)$$

where Q is the inlet discharge (m^3/s); A_o is the cross-sectional area of water at the upstream end of the field (m^2); and, σ_y and σ_z are the averaging coefficients (shape factors).

Using Eqs. 2 and 3, together with the information of two points located at the midpoint and endpoint of the field, the two-point method allows for the calculation of coefficients r , a , and k as determined by Eqs. 4 and 5.

$$r = \frac{\log(1/2)}{\log(t_{L/2}/t_L)} \quad (4)$$

where $t_{L/2}$ is the time required for water to reach the midpoint of a field (min), and t_L is the time needed for water to reach the endpoint of the same field (min).

$$a = \frac{\log(V_L/V_{L/2})}{\log(t_L/t_{L/2})} \quad \& \quad k = \frac{V_L}{\sigma_z t_L^a} \quad (5)$$

where $V_{L/2}$ is the volume of water infiltrated to the midpoint of the field (m^3) and V_L is the volume of water infiltrated to the endpoint of the field (m^3).

σ_z parameter can be computed through integrating Eq. 6, or using the approximate formulation offered by Kiefer (1965).

$$\begin{aligned} \sigma_z &= \frac{\int_0^L kt_x^a dx}{kt_L^a} \\ &= r\beta(r, a+1) \xrightarrow{\text{Kiefer(1965)}} \sigma_z \\ &\cong \frac{a+r(1-a)+1}{(1+a)(1+r)} \end{aligned} \quad (6)$$

where β is the beta function.

In this study, a point was chosen as the midpoint where the infiltration opportunity was equal to the mean infiltration opportunity time of the advance phase. This time was determined when the duration of the advance phase was at its average value.

$$\overline{t_n} = \overline{t_L} - \overline{t_x} = \overline{t_L} - \overline{t_x} \quad (7)$$

where $\overline{t_n}$ is the average infiltration opportunity time (min); $\overline{t_x}$ is the average advance time along the field (min).

According to Eq. 2, the relationship of t_x is as follows.

$$x = pt_x^r \quad \rightarrow \quad t_x = (x/p)^{1/r} \quad (8)$$

By calculating the average of Eq. 8, the value of \bar{t}_x is obtained as follows:

$$\bar{t}_x = \frac{\int_0^L t_x dx}{L} = \frac{\int_0^L \left(\frac{x}{p}\right)^{\frac{1}{r}} dx}{L} \quad (9)$$

$$= \frac{1}{p\left(\frac{1}{r}\right)_L} \times \frac{x\left[\left(\frac{1}{r}\right)+1\right]}{1 + \left(\frac{1}{r}\right)} \Big|_0^L$$

$$\bar{t}_x = \frac{1}{p\left(\frac{1}{r}\right)_L} \times \frac{rL\left[\left(\frac{1}{r}\right)+1\right]}{1+r} \quad (10)$$

$$= \frac{r}{r+1} \left(\frac{L}{p}\right)^{\left(\frac{1}{r}\right)} \rightarrow \bar{t}_x = \frac{r}{r+1} t_L$$

By inserting Eq. 10 in Eq. 8, relation \bar{x} becomes as follows:

$$\bar{x} = p \left(\frac{r}{r+1}\right)^r t_L^r \rightarrow \bar{x} = L \left(\frac{r}{r+1}\right)^r \quad (11)$$

where \bar{x} is the distance from the upstream end of the field where the average infiltration opportunity time occurs in the advance phase (m).

The value of r can be calculated by employing the water advance data analysis

and curve fitting techniques. Subsequently, after finding the value of r , the values of two parameters \bar{t}_x and \bar{x} can be computed using Eqs. 10 and 11, respectively. Finally, the infiltration coefficients of the Kostiakov-Lewis equation will be determined by utilizing the water advance information at the end of the field (L, t_L), along with using \bar{x} and \bar{t}_x . Panahi et al. (2021) suggested the midpoint as the point where half of the average infiltration opportunity time occurred during the advance time. Also, they obtained the average advance time through the geometric mean of the advance time of the stations.

2.2. Field Data

Data from three closed-end irrigation systems, Vahedi 2, Vahedi 3 and Orooj, were used for the evaluation, with the crops grown being alfalfa and the stations set up at 10-meter intervals. The inflow to the farms was measured with the Washington State College (WSC) flume type 3 and the geometric characteristics of the farms are provided in Table 1.

Table 1. Geometric characteristics of Orooj, Vahedi 2 and Vahedi 3.

Field Name	Irrigation System Type	Farm Length (m)	Farm Width (m)	Longitudinal Slope (%)	Number of Evaluated Irrigation	Downstream Condition
Orooj	Border	126	6.75	0.45	3	Blocked
Vahedi 2	Border	109	3	0.28	2	Blocked
Vahedi 3	Border	107.5	3	0.27	2	Blocked

The relative error index and the Root Mean Square Deviation (d_{RMS}) index were employed to quantify the accuracy of the water advance relationships attributed to the method proposed in this research (TR) and the two-point method of Elliott and Walker (1982) (EW). The d_{RMS} index relationship utilized by Elliott and Walker (1982) to evaluate the water advance relationships is as follows:

$$d_{RMS} = \sqrt{\sum_{i=1}^n (t_{i,P} - t_{i,O})^2} \quad (12)$$

where $t_{i,O}$ is the actual advance time (min) at the i^{th} station; and, $t_{i,P}$ is the estimated advance time at the i^{th} station.

3. Results and Discussion

By employing the data on the water advance along the field and conducting an iterative approach (employing the Excel solver option), the value of the exponent coefficient (r) was computed so as to minimize the discrepancies between the computed water advance relationship and the experimental field data. Subsequently, the average advance time and the distance of the midpoint from the upstream end of the field were determined

by employing Eqs. 10 and 11. Also, in the two-point method of Elliott and Walker (1982), the value of the coefficient r was derived by utilizing data from two points at the midpoint $(L/2, t_{L/2})$ and endpoint (L, t_L) of the field. Table 2 presents the values of the coefficient r and the midpoint location derived from the TR and EW methods.

Table 2. The values of the coefficient r and the midpoint location derived from the TR and EW methods in the study farms.

Field Name	Irrigation No.	$t_{L/2}$ (min)	\bar{t}_x (min)	Midpoint Location (m)		r	
				TR (\bar{x})	EW (L/2)	TR	EW
Vahedi 2	1	32.2	32.6	57.4	54.5	0.77	0.82
	2	40.5	44.3	63.4	54.5	0.48	0.57
Vahedi 3	1	34.9	34.7	55.9	53.8	0.82	0.87
	2	23.1	26.7	58.8	53.8	0.64	0.64
Orooj	1	94.4	98.9	69.3	63.0	0.63	0.69
	2	79.3	82.4	66.2	63.0	0.78	0.80
	3	68.9	80.6	67.8	63.0	0.69	0.66

Table 3 and Figure 1 show the accuracy of the water advance relationships derived from TR and EW methods.

Table 3. The values of d_{RMS} and relative error indicators of the water advance relationships derived from TR and EW methods in the study farms.

Field Name	Irrigation No.	Error (%)		d_{RMS} (min)	
		TR	EW	TR	EW
Vahedi 2	1	6.5	8.9	6.8	7.9
	2	25.8	15.1	26.3	31.9
Vahedi 3	1	3.7	8.6	4.8	6.6
	2	13.8	14.0	7.6	7.6
Orooj	1	15.7	8.9	42.3	48.3
	2	6.1	6.7	15.7	16.5
	3	8.6	10.6	17.0	19.2

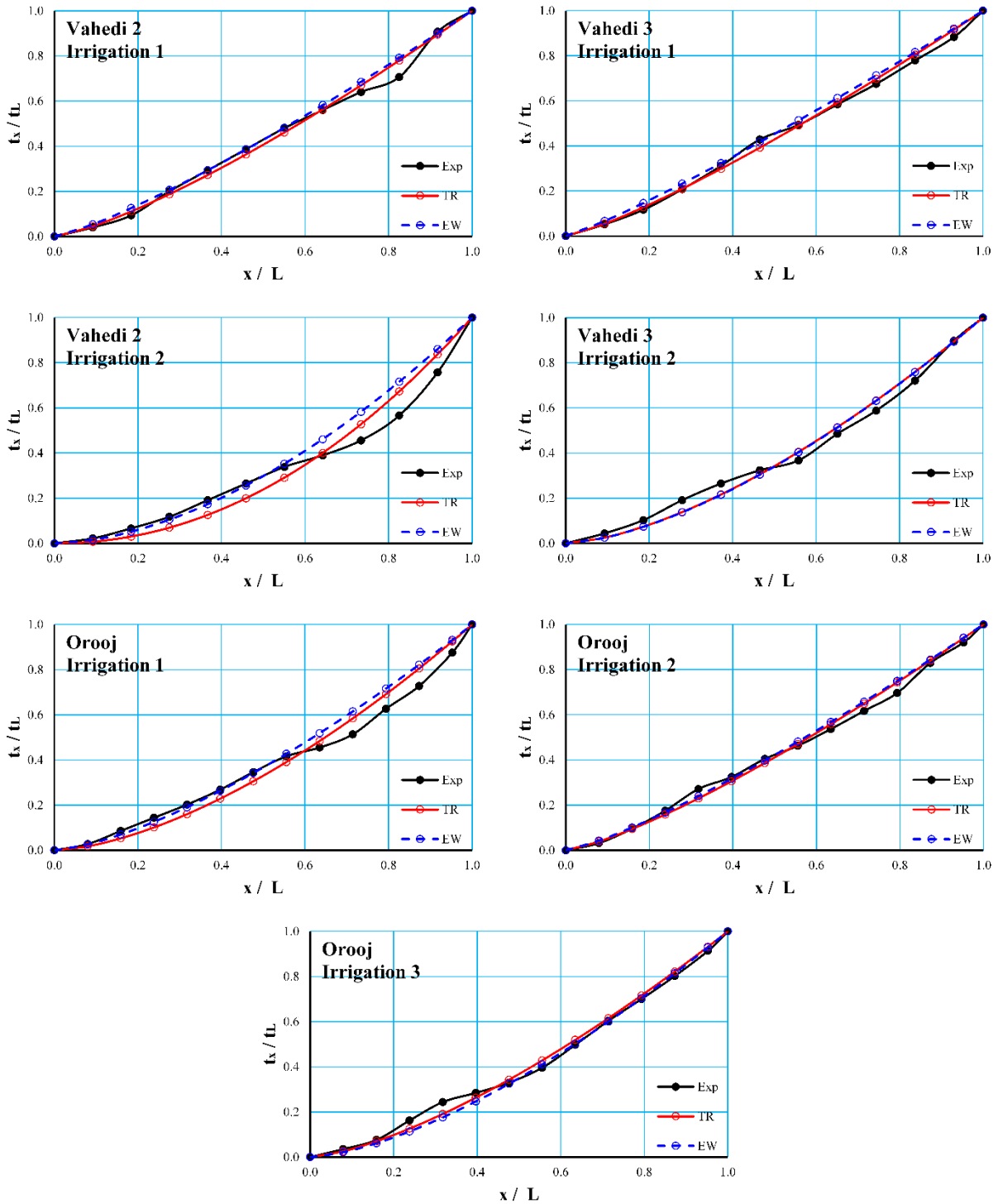


Fig 1. The changes in the water advance relationships derived from the TR and EW methods for different irrigations.

Table 3 revealed that, in the majority of irrigation events, the TR method had a lower relative error index, implying a more accurate water advance relationship. For the Vahedi 2 farm's second irrigation and the Orooj farm's

first irrigation, the EW method obtained a water advance relationship with a lower relative error, as evidenced by Figure 1 and the associated graphs. According to the graphs related to these irrigations, it is

evident that the EW's water advance relationship and the field data in the first half of the field correlate closely, whereas the TR's water advance relationship deviates from the field data to a greater degree. In the last half of the field, the TR water advance relationship has a smaller deviation from the field data. At the beginning stations of the field, where the advance time is short, even small discrepancies between the advance curve and the observed field data can lead to considerably large errors, whereas they will result in significantly fewer errors when the same discrepancies occur in the latest stations of the field. Therefore, the relative error index is not a suitable index to compare water advance relations. Therefore, in this study, in addition to the relative error index, the d_{RMS} index was also used.

Based on the d_{RMS} index, it is evident that the TR method holds a more accurate water advance relationship than the EW method. However, the superiority of the TR method over the EW method is not vastly apparent for most irrigation events. Consequently, the d_{RMS} index implies that both approaches display satisfactory accuracy.

Considering the importance of the water advance relationship in determining infiltration coefficients and calculating infiltration depth, these relationships were used to determine infiltration coefficients and average infiltration depth. Therefore, using these water advance relationships and the two-point method, the values of the coefficients of the Kostiakov-Lewis infiltration relationship were determined, the results of which are presented in Table 4.

Table 4. The values of the coefficients of the Kostiakov-Lewis infiltration relationship derived from the TR and EW water advance relationships for the studied farms.

Field Name	Irrigation No.	a		k (cm / hr ^{a})		f_0 (cm/hr)
		TR	EW	TR	EW	
Vahedi 1	1	0.297	0.226	22.17	22.66	1.398
	2	0.642	0.514	13.03	14.43	
Vahedi 2	1	0.223	0.142	24.32	23.51	
	2	0.503	0.517	16.92	16.98	
Orooj	1	0.383	0.300	12.02	13.23	0.924
	2	0.177	0.148	8.20	8.33	
	3	0.310	0.361	10.03	9.61	

Then, using the infiltration relations presented in Table 4 and the average infiltration opportunity, the average infiltration depth was calculated. Using the

relative error index, calculated infiltration depth values were compared with the actual (farm) average infiltration depth, the results of which are presented in Table 5.

Table 5. The average error of the infiltration depth of the infiltration relations derived by the TR and EW method for the studied fields.

Field Name	Irrigation No.	Infiltration Depth's Error (%)	
		TR	EW
Vahedi 1	1	0.0	6.2
	2	7.1	6.7
Vahedi 2	1	2.5	5.7
	2	1.8	1.2
Orooj	1	2.4	2.0
	2	5.5	6.4
	3	0.6	1.7

Table 5 indicates that the accuracy of the TR method (average error of 2.4% for the average infiltration depth) is higher than that of the EW method due to the randomization of coefficients in the infiltration relationship caused by the use of two fixed location points in the EW method.

4. Conclusion

The main objective of this study is to identify the ideal location for the midpoint in the two-point method. The proposed midpoint location is the location at which the average infiltration opportunity time in the water advance phase occurs. To determine the location of this point, the exponential equation for the water advance relationship was taken into consideration, and its exponent coefficient was obtained iteratively in order to minimize the mean error between the calculated and the field data of the water advance. Then, the coefficients of the Kostiakov-Lewis infiltration relationship were obtained by applying the water advance relationships established in this study and the two-point method of Elliott and Walker (1982). To evaluate the accuracy of the derived water advance relationships and infiltration relationships, field data were employed in comparison. The results showed that the present method of midpoint location determination is more precise than the two-point method.

Data Availability

The data used to support the findings of this study is available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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