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A New Method for Determining Coefficients of Kostiakov Infiltration Relationship

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Abstract

In this study, a new advance relation (TR) is presented, which has only one constant coefficient. To determine the value of this coefficient, the water advance information at the midpoint and endpoint along the field is used. Field data from six irrigation events is used to evaluate this relationship and compare it with the Elliott and Walker's (EW) exponential advance relationship. EW and TR advance relationships are compared using the relative error, Root Mean Square Deviation (RMSD), and Nash-Sutcliffe Efficiency (NSE) indices. The result of this comparison show that the two advance relationships have equal accuracy in a number of irrigation events, and the EW advance relationship has more accuracy in other events. Then using the TR advance relationship, a new method was presented to determine the subsurface storage coefficient in different lengths of the field and as a result to determine the coefficients of the Kostiakov infiltration relationship. The error-index for the average infiltration depth was used to compare the infiltration relations obtained from the EW and TR methods. The results of this comparison showed that the infiltration relationships of the two methods had equal accuracy in numerous irrigation events, and in some cases, the infiltration relationships obtained from the TR method are more accurate.

1. Introduction

Traditional surface irrigation methods remain widely used around the world. Poor management and unsuitable design can reduce water efficiency (Merriam, 1977; Seyedzadeh *et al*., 2022a). Consequently, modifying surface irrigation systems can improve performance indicators (Seyedzadeh *et al*., 2019). A key element in the design and assessment of surface irrigation systems is infiltration (Fig. 1) (Seyedzadeh *et al*., 2022b; Walker and Skogerboe, 1987). Kostiakov (1932) suggested a set of models to explain the rate of infiltration. The Kostiakov equation is the most widely utilized model for surface irrigation due to its simplicity and its ability to approximate a variety of infiltration data. The Kostiakov infiltration equation is as follows:

$$i = kt^{a} \tag{1}$$

where i is the cumulative infiltration (m), t is the time of infiltration (min), and, k (m/min^{a}) and a (dimensionless) are the constant coefficient.

Various techniques have been developed to calculate the coefficients of the Kostiakov equation based on surface irrigation

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observations. Christiansen *et al*. (1966) used a volume balance model to determine k and a based on water advance. Elliott and Walker (1982) applied the same technique to calculate infiltration characteristics. Norum and Gray (1970) employed curve-fitting techniques to obtain k and a from advance data for border-dike irrigation. Elliott *et al*. (1983) utilized matching techniques with dimensionless advance curves to find the coefficients of the Kostiakov equation. Wallender and Sirjani (1988) used advance data to predict the mean values and variance of k and a. Reddell and Latartue (1986) and Reddell and Latortue (1988) proposed techniques to get the coefficients of various modified forms of Kostiakov equation using advance data. Smerdon *et al*. (1988) presented a useful and reliable technique for estimating values of *k* and *a* from field data, which was then evaluated by Blair and Smerdon (1988). Also in the recent years, researchers such as Seyedzadeh *et al*. (2020a), Seyedzadeh *et al*. (2020b), Panahi *et al*. (2021), and Panahi *et al*. (2022) have presented new methods for determining the coefficients of the infiltration relationship, which have a good accuracy.

Elliott and Walker (1982) demonstrated their own two-point method, which relies on the volume balance relationship. Specifically, they assumed that the water advance along the field follows an exponential relationship (Eq. 2).

$$x = pt_x^{\ r} \tag{2}$$

where x is the distance of water advance from upstream end of the field (m), t_x is the time of the water advance from the upstream end of the field to x location (min), and p and r are the constant coefficients (dimensionless).

The accurate mathematical expression of water advance along the field will increase the accuracy in determining the volume of surface stored water and as a result, accurately determine the infiltration relationship coefficients (Emamgholizadeh *et al*., 2022).

In this study, a new water advance relationship with more accuracy and flexibility is presented, and using it, a new method to determine coefficients of the Kostiakov infiltration relationship is presented.

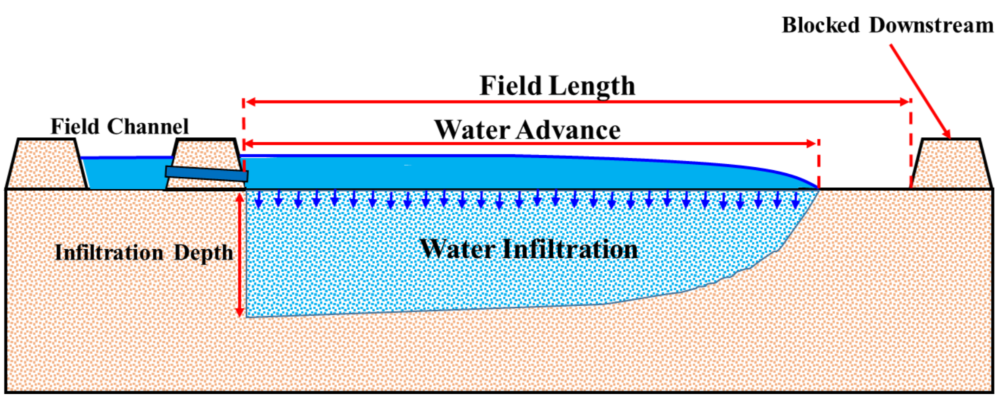

Figure. 1. Sketch of water infiltration along the field.

2. Martials and Methods

2.1. Theory Background

The volume balance relationship for determining the coefficients of the Kostiakov infiltration relationship is as follows:

$$V_x = \sigma_z k t_x^{\ a} = \frac{Qt_x}{x} - \sigma_y A_o \tag{3}$$

where Q is the inlet discharge (m^3/s); A_o is the flow cross-section area at the upstream end of the field (m^2); and σ_y and σ_z are the averaging coefficients (shape factors).

In Eq. 3, the value of the σ_y coefficient is assumed to be 0.77, and the value of the σ_z coefficient can be determined using either the integral relationship or the approximate relationship proposed by Kiefer (1965):

$$\sigma_z = \frac{\int_0^L k t_x^{\,a} dx}{k t_L^{\,a}} = r\beta(r, a+1) \xrightarrow{Kiefer(1965)} \sigma_z \cong \frac{a + r(1-a) + 1}{(1+a)(1+r)} \quad (4)$$

where t_L is the water arrival time from the upstream end to the downstream end of the field (min), L is the filed length (m), and β is the beta function.

In the two-point method, using the information of the midpoint and the endpoint of the field and using Eqs. 5 and 6, the values of coefficients r, a, and k can be determined.

$$r = \frac{\log(1/2)}{\log\left(t_{\frac{L}{2}} / t_L\right)} \quad (5)$$

$$a = \frac{\log(V_L / V_{L/2})}{\log\left(t_L / t_{L/2}\right)} \quad \& \quad k = \frac{V_L}{\sigma_z t_L^{\,a}} \quad (6)$$

where $t_{L/2}$ is the water arrival time from the upstream end to the midpoint of the field (min), $V_{L/2}$ is the volume of water infiltrated until the water reaches the midpoint of the field (m^3), and V_L is the volume of water infiltrated until the water reaches the endpoint of the field (m^3).

In this study, the water advance relationship along the field is considered as follows:

$$x = \frac{L}{2}\left[\frac{t_x}{t_L} + \left(\frac{t_x}{t_L}\right)^r\right] \quad (7)$$

In Eq. 7, if the advance time of t_x is set equal to zero, the water advance (x) is equal to zero, and if it is placed equal to t_L, the water advance is equal to L. The value of r is determined using the midpoint information ($L/2$, $t_{L/2}$) as follows:

$$\frac{L}{2} = \frac{L}{2}\left[\frac{t_{\frac{L}{2}}}{t_L} + \left(\frac{t_{\frac{L}{2}}}{t_L}\right)^r\right]$$
$$\left(\frac{t_{\frac{L}{2}}}{t_L}\right)^r = \left(1 - \frac{t_{\frac{L}{2}}}{t_L}\right) \quad (8)$$
$$r = \frac{ln\left(1 - \frac{t_{\frac{L}{2}}}{t_L}\right)}{ln\left(\frac{t_{\frac{L}{2}}}{t_L}\right)}$$

To obtain σ_z , the total volume of water infiltration during the advance is calculated and then divided by the amount of infiltration at the upstream end of the field. To calculate it, first, the value of dx in terms of dt_x is determined using Eq. 7 as follows:

$$\frac{dx}{dt_x} = \frac{L}{2}\left(\frac{1}{t_L} + \frac{r t_x^{\,r-1}}{t_x^{\,r}}\right) \quad (9)$$

The general relationship to determine the σ_z coefficient is as follows:

$$\sigma_z = \frac{\int_0^m k(t_m - t_x)^a \, dx}{m k t_m^{\,a}} \quad (10)$$

where m is the location of the water advance at time t_m, so that $0 \le x \le m$.

In Eq. 10, the function located above the fraction line is solved as follows:

$$\int_0^m k(t_m - t_x)^a \, dx = \int_0^{t_m} \frac{kL}{2}(t_m - t_x)^a \left(\frac{1}{t_L} + \frac{r t_x^{\,r-1}}{t_L^{\,r}}\right) dt_x$$
$$= \frac{kL}{2}\int_0^{t_m}\left[\frac{(t_m - t_x)^a}{t_L} + \frac{r t_x^{\,r-1}(t_m - t_x)^a}{t_L^{\,r}}\right] dt_x \quad (11)$$

In the above relationship, a variable change can be considered as follows:

$$\frac{t_x}{t_m} = u \quad \rightarrow \quad dt_x = t_m du \quad (12)$$

By inserting Eq. 12 in Eq. 11, and integrating Eq. 11, it will be as follows:

$$-\frac{kL}{2t_L}\frac{(t_m-t_x)^{a+1}}{a+1}\Bigg|_0^{t_m}+\frac{kLr}{2t_L^{\,r}}\int_0^1 t_m^{\,a+r}(1-u)^a u^{r-1}du = \frac{kLt_m^{(a+1)}}{2t_L(a+1)}+\frac{kLr}{2t_L^{\,r}}t_m^{(a+r)}\beta(r,a+1) \quad (13)$$

By inserting Eq. 13 in Eq. 10 and simplifying it, the equation for determining σ_z becomes as follows:

$$\sigma_z=\frac{\frac{kL}{2}\left[\frac{t_m^{(a+1)}}{t_L(a+1)}+\frac{r}{t_L^{\,r}}t_m^{(a+r)}\beta(r,a+1)\right]}{mkt_m^{\,a}} \qquad \sigma_z=\frac{L}{2m}\left[\frac{t_m}{t_L(a+1)}+r\left(\frac{t_m}{t_L}\right)^r\beta(r,a+1)\right] \quad (14)$$

According to Eq. 14, it is clear that the value of σ_z depends on the value of *m*. Therefore, the value of σ_z for advancing to the midpoint and endpoint can be determined using Eqs. 15 and 16, respectively.

$$\sigma_z(\frac{L}{2})=\left[\frac{t_{L/2}}{t_L(a+1)}+r\left(\frac{t_{L/2}}{t_L}\right)^r\beta(r,a+1)\right] \quad (15)$$

$$\sigma_z(L)=\frac{1}{2}\left[\frac{1}{(a+1)}+r\beta(r,a+1)\right] \quad (16)$$

According to Eq. 3, the water infiltrated volume in the water advance to the midpoint and endpoint of the field can be calculated using Eqs. 17 and 18, respectively:

$$V_{L/2}=\left[\sigma_z(\frac{L}{2})\right]kt_{L/2}^{\,a}=\frac{2Qt_{L/2}}{L}-\sigma_y A_o \quad (17)$$

$$V_L=\left[\sigma_z(L)\right]kt_L^{\,a}=\frac{Qt_L}{L}-\sigma_y A_o \quad (18)$$

By dividing from Eqs. 17 and 18, the value of coefficients *a* and *k* is determined as follows:

$$\frac{\sigma_z(L)}{\sigma_z(L/2)}\left(\frac{t_L}{t_{L/2}}\right)^a=\frac{V_L}{V_{L/2}} \qquad a=\frac{\ln\left[\frac{\sigma_z(L/2)}{\sigma_z(L)}\times\frac{V_L}{V_{L/2}}\right]}{\ln\left(t_L/t_{L/2}\right)} \quad (19)$$

$$k=\frac{V_L}{\sigma_z(L)t_L^{\,a}} \quad (20)$$

2.2. Field data

Three border-irrigated fields Vahedi 1, Vahedi 2, and Vahedi 3 from the Zarrineh Rood irrigation and drainage network in western Iran were evaluated. The location of the studied farms is shown in Fig. 2. During two irrigation events, the borders of the Vahedi field were stationed at 10-meter intervals. The flow rate of the borders was measured by using the Type 3 Washington State College (WSC) flume. The geometric features of the experimental borders are shown in Table 1.

Table 1. Geometric characteristics of the experimented borders.

Field name	Border No.	Border length (m)	Border width (m)	Longitudinal slope (%)	Number of evaluated irrigations	Downstream condition
Vahedi	1	109	3	0.28	2	Blocked (no runoff)
	2	109	3	0.27	2	
	3	107.5	3	0.26	2	

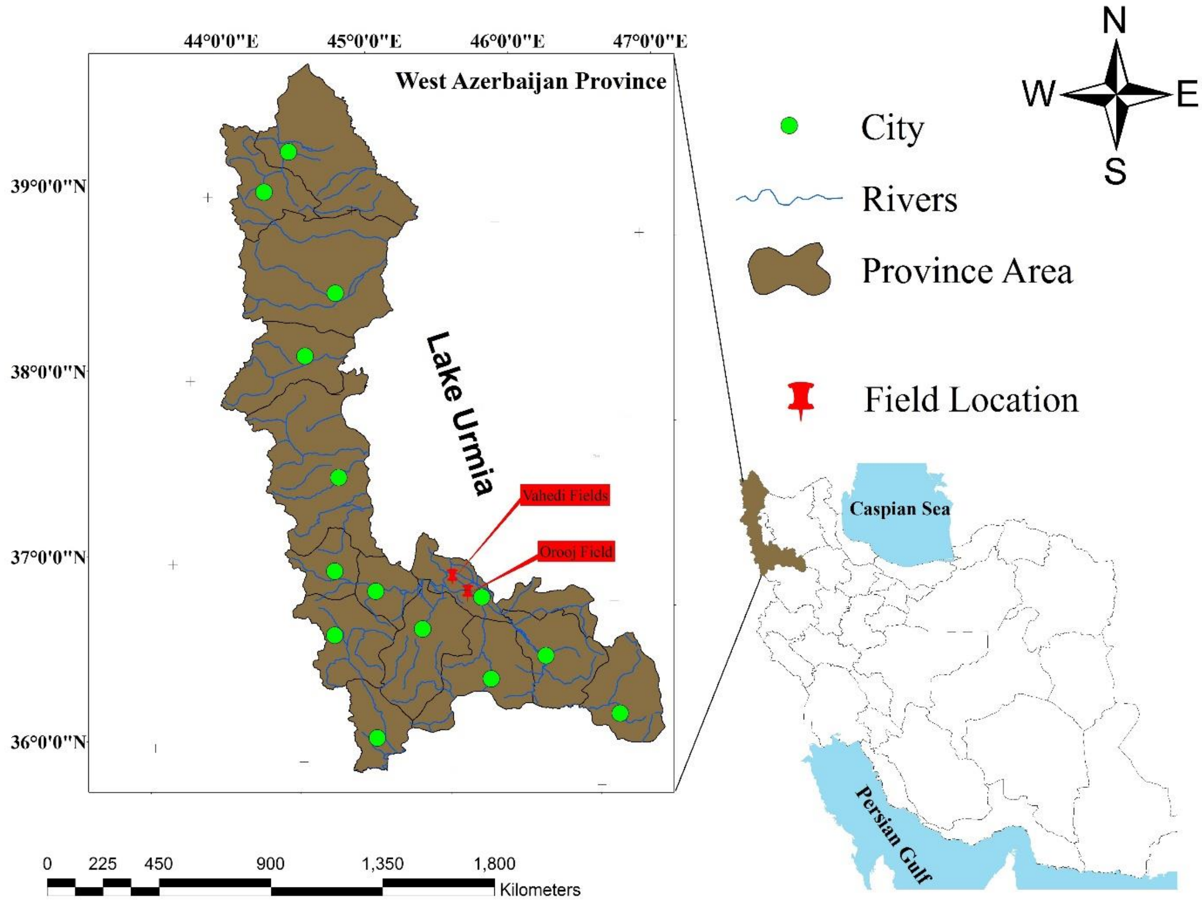

Figure. 2. Location of studied farms.

2.3. Evaluation indicators

To evaluate the water advance relationship presented in this study (TR), using the information of the midpoint and endpoint, the coefficient r related to the water advance relationship of this study and the water advance relationship of Elliott and Walker (1982) (EW) is calculated. Then using the relative error, the Root Mean Square Deviation (*RMSD*), and the Nash-Sutcliffe Efficiency (*NSE*) indices the accuracy of the water advance relations is evaluated.

The relations of the *RMSD* and *NSE* indices are as Eqs. 21 and 22, respectively.

$$RMSD = \sqrt{\sum_{i=1}^{n}\left(x_{i,P} - x_{i,O}\right)^2} \tag{21}$$

$$NSE = 1 - \frac{\sum_{i=1}^{n}\left(x_{i,O} - x_{i,P}\right)^2}{\sum_{i=1}^{n}\left(x_{i,O} - \overline{x_{i,O}}\right)^2} \tag{22}$$

where *i* is the counter of the stations located along the field, n is the total number of stations, $\overline{x_{i,O}}$ is the average distance of the water advance from the upstream end of the field, and $x_{i,O}$ and $x_{i,P}$ are the experimental and calculated distance of the water advance from the upstream end of the field, respectively.

The Nash-Sutcliffe index can vary from an infinite negative number to one. If it is equal to one, then the observed and predicted data will be perfectly matched (Moriasi *et al.*, 2007). If the index is above 0.75, then the results will be considered to be a good fit. On the other hand, if the value lies between 0.36 and 0.75, the predicted results will be of a mediocre to good quality (Motovilov *et al*., 1999).

3. Results and Discussion

Using the TR and EW methods, the value of the *r* coefficient was calculated for each of the irrigation events and the advance relationships obtained from these methods were evaluated using the relative error, *RMSD*, and *NSE* indices. The results of the evaluation of the advance relationships obtained from the TR and EW methods are presented in Table 2.

Table 2. Results of the evaluation of TR and EW advance relationships for the studied farms.

Border name	Irrigation No.	*r*		Error (%)		*RMSD* (m)		*NSE*	
		TR	EW	TR	EW	TR	EW	TR	EW
Vahedi 1	1	0.69	0.83	6.9	7.8	18.3	18.2	0.85	0.85
	2	0.23	0.52	21.5	7.2	24.2	14.4	0.80	0.88
Vahedi 2	1	0.66	0.82	4.7	5.9	10.4	10.6	0.91	0.91
	2	0.29	0.57	19.6	8.8	25.3	18.8	0.79	0.84
Vahedi 3	1	0.76	0.87	6.0	6.5	8.7	8.9	0.93	0.92
	2	0.38	0.64	17.7	11.7	17.8	13.1	0.85	0.89

According to Table 2, based on the relative error and *RMSD* indices, in the first irrigations, both methods had the same accuracy. However, in the second irrigation events, the EW advance relationships were more accurate than the TR advance relationships. Of course, the relative error index is not a suitable index for comparing two advance relationships. Because this index shows the least difference with a large error in the initial stations, while the accurate determination of the end stations of the field is more important (Walker and Skogerboe, 1987). Therefore, the *NSE* index was also used to better compare these two advance relationships. The comparison of the methods based on this index shows the high ability of both methods and there is no noticeable difference between these two methods.

Due to the greater importance of determining the infiltration depth in surface irrigation design and also the application of the advance relationship in determining the infiltration coefficients and as a result the infiltration depth, these relationships were used to determine the infiltration coefficients. In Table 3, the coefficients of the Kostiakov infiltration relationship derived using the EW and TR methods are presented.

Table 3. Coefficients of Kostiakov infiltration relationship obtained by EW and TR methods for the studied farms.

Border name	Irrigation No.	TR		EW	
		a	*k* (cm/hr^a)	*a*	*k* (cm/hr^a)
Vahedi 1	1	0.229	24.65	0.225	24.70
	2	0.758	13.59	0.692	13.84
Vahedi 2	1	0.259	23.02	0.254	23.06
	2	0.602	15.52	0.555	15.92
Vahedi 3	1	0.171	24.88	0.169	24.92
	2	0.577	18.44	0.542	18.37

Using the infiltration relationships obtained from EW and TR methods and also using the average water infiltration opportunity time in the studied fields, the average infiltration depth in each irrigation event was calculated. Using the relative error index of infiltration depth, the calculated average infiltration depth obtained from each of the methods was compared with the actual average infiltration depth. Table 4 shows the results of this comparison.

Table 4. Evaluation results of infiltration relationships obtained from EW and TR methods for the studied farms.

Border name	Irrigation No.	Infiltration depth's error (%)	
		TR	EW
Vahedi 1	1	4.6	4.7
	2	0.0	1.8
Vahedi 2	1	7.8	8.0
	2	7.8	6.3
Vahedi 3	1	8.3	8.3
	2	0.4	1.5

According to Table 4, it can be seen that in the first irrigation events, the advance relationships of both methods had the same accuracy, so the relative error of their infiltration depth also has the same accuracy. But in the second irrigation events, despite the fact that the EW advance relationship had a lower error percentage, the infiltration relationships obtained from the TR method are more accurate in estimating the average infiltration depth.

4. Conclusion

The main objective of this study (TR) was to present a new water advance relationship for predicting water advance along the field. In the two-point method (EW) presented by Elliott and Walker (1982), a relationship was used to determine the sub-surface storage coefficient both in determining the water-infiltrated volume to the midpoint and in determining the water-infiltrated volume to the endpoint of the field. In this study, using the presented water advance relationship, a relationship was presented to determine the subsurface storage coefficient for any desired field length. In the following, using the volume balance relationship, the two-point method, and the subsurface storage coefficient relationships for the midpoint and endpoint of the field, a new method was presented to determine the coefficients of the Kostiakov infiltration relationship. The water advance relationships obtained from EW and TR methods as well as the infiltration relationships obtained from these methods were compared.

Data Availability

The data used to support the findings of this study is available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

References

Blair, A. and Smerdon, E., 1988. Infiltration from irrigation advance data. II: Experimental. Journal of irrigation and drainage engineering, 114(1): 18-30.

Christiansen, J., Bishop, A., Kiefer, F., and Fok, Y.-S., 1966. Evaluation of intake rate constants as related to advance of water in surface irrigation. Transactions of the ASAE, 9(5): 671-0674.

Elliott, R. and Walker, W., 1982. Field evaluation of furrow infiltration and advance functions. Transactions of the ASAE, 25(2): 396-0400.

Elliott, R.L., Walker, W.R., and Skogerboe, G.V., 1983. Infiltration parameters from furrow irrigation advance data. Transactions of the ASAE, 26(6): 1726-1731.

Emamgholizadeh, S., Seyedzadeh, A., Sanikhani, H., Maroufpoor, E., and Karami, G., 2022. Numerical and artificial intelligence models for predicting the water advance in border irrigation. Environment, Development and Sustainability, 24(1): 558-575.

Kiefer, F., 1965. Average depth of absorbed water in surface irrigation. Special Publication. Civil Engineering Department, Utah State University, Logan, Utah.

Kostiakov, A.N., 1932. On the dynamics of the coefficient of water-percolation in soils and on the necessity of studying it from a dynamic point of view for purposes of amelioration. Trans. 6th Cong. International. Soil Science, Russian Part A: 17-21.

Merriam, J.L., 1977. Efficient irrigation. California Polytechnic State University. San Luis Obispo, California.

Moriasi, D.N. et al., 2007. Model evaluation guidelines for systematic quantification of accuracy in watershed simulations. Transactions of the ASABE, 50(3): 885-900.

Motovilov, Y.G., Gottschalk, L., Engeland, K., Rodhe, A., 1999. Validation of a distributed hydrological model against spatial observations. Agricultural and Forest Meteorology, 98: 257-277.

Norum, D.I. and Gray, D.M., 1970. Infiltration equations from rate-of-advance data. Journal of the Irrigation and Drainage Division, 96(2): 111-119.

Panahi, A., Alizadeh-Dizaj, A., Ebrahimian, H., and Seyedzadeh, A., 2022. Estimating Infiltration in Open-ended Furrow Irrigation by Modifying Final Infiltration Rate. Irrigation and Drainage, 71(3): 676-686.

Panahi, A., Seyedzadeh, A., and Maroufpoor, E., 2021. Investigating the midpoint of a two-point method for predicting advance and infiltration in surface irrigation. Irrigation and Drainage, 70(5): 1095-1106.

Reddell, D. and Latartue, H., 1986. Evaluation of furrow surface storage and the Kostiakov infiltration parameters using irrigation advance data. American Society of Agricultural Engineers. Microfiche collection (USA).

Reddell, D. and Latortue, H., 1988. Estimating the three-parameter kostiakov infiltration function for furrow irrigation with advanced data. American Society of Agricultural Engineers (Microfiche collection)(USA).

Seyedzadeh, A., Khazaee, P., Siosemardeh, A., and Maroufpoor, E., 2022a. Irrigation management evaluation of multiple irrigation methods using performance indicators. ISH Journal of Hydraulic Engineering, 28(3): 303-312.

Seyedzadeh, A., Panahi, A., and Maroufpoor, E., 2020a. A new analytical method for derivation of infiltration parameters. Irrigation Science, 38: 449-460.

Seyedzadeh, A., Panahi, A., Maroufpoor, E., and Merkley, G.P., 2022b. Subsurface shape factor for surface irrigation hydraulics. Irrigation and Drainage, 71(3): 687-696.

Seyedzadeh, A., Panahi, A., Maroufpoor, E., and Singh, V.P., 2019. Development of an analytical method for estimating Manning's coefficient of roughness for border irrigation. Irrigation Science, 37: 523-531.

Seyedzadeh, A., Panahi, A., Maroufpoor, E., Singh, V.P., and Maheshwari, B., 2020b. Developing a novel method for estimating parameters of Kostiakov–Lewis infiltration equation. Irrigation Science, 38: 189-198.

Smerdon, E., Blair, A., and Reddell, D., 1988. Infiltration from irrigation advance data. I: Theory. Journal of irrigation and drainage engineering, 114(1): 4-17.

Walker, W. and Skogerboe, G., 1987. Surface irrigation: theory and practice prentice–hall. Inc. Englewood cliffs. New Jersey.

Wallender, W. and Sirjani, F., 1988. Stochastic infiltration from advance in furrows. American Society of Agricultural Engineers (Microfiche collection)(USA).